

The thermoelement as thermoelectric power generator: Effect of leg geometry on the efficiency and power generation

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ABSTRACT

Thermoelectric power generators are one of the promising clean energy resources with the cost effective operation despite the low device efficiency. Investigation into device efficiency improvement is necessary for the practical applications. Consequently, in the present study, a theoretical analysis of thermoelectric power generator is carried out and influence of thermoelectric leg geometry on the device efficiency and the power generation is formulated. The geometric configuration of the legs in the device is associated with the shape parameter and incorporated in the analysis. The influence of the shape parameter on the device efficiency and power generation is examined for various temperature and external load resistance ratios. It is found that increasing or decreasing of the shape parameter (μ) has a favorable effect on the device efficiency; however, the shape parameter (μ) has an adverse effect on the thermoelectric power generation.

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1. Introduction

The requirement of clean energy and the technological developments lead to extensive research into the renewable energy resources. One of the promising clean energy sources is the thermoelectric power generators. Since these devices do not have the moving parts, the operation and maintenance of thermoelectric power generators are easy and less costly. The efficiency of thermoelectric generator is highly depending on the operating temperatures, the figure of merit, and design configuration including the external load parameter of the device. In most of the power generation applications, the figure of merit is on the order of 1 resulting in low device thermal efficiency. Although the research into thermoelectric material development for high figure of merit is progressing, the maximum device thermal efficiency is still on the order of less than 10%. Since the design configuration including geometric arrangements of the thermoelectric legs influences the thermal efficiency, investigation into leg geometry on the thermoelectric power generation and the device thermal efficiency becomes essential. Many aspects of thermoelectrics including energy conversion, performance optimization, thermoelectric material characterization, and various applications have been studied extensively in the literature [1–14].

Considerable research studies were also carried out to examine thermoelectric power generation and device efficiency. Design and thermal analysis of solar thermoelectric power generation system

were carried out by Vatcharasathien et al. [15]. They considered truncated parabolic collectors with a flat receiver, conventional flat-plate collectors, and thermoelectric power generator modules in the performance analysis. The optimization study for the waste heat thermoelectric generator system was carried out by Gou et al. [16]. They indicated that the promising potential was present to use a thermoelectric generator for a low-temperature waste heat recovery, especially in the industrial fields. The system performance of the thermoelectric generator was investigated by Xiao et al. [17]. They introduced a mathematical model for the thermoelectric generator performance using the finite time thermodynamic analysis. Thermoelectric power conversion from heat re-circulating combustion systems was examined by Weinberg et al. [18]. The findings revealed that the efficiency of thermoelectric devices could be improved for certain arrangements of the locations of the devices around the heat transferring surface. The model study in relation to design of a thermoelectric power generator maximizing the performance of the device was carried out by Crane and Bell [19]. They introduced the gradient based optimization technique to incorporate the interactions between various design variables and parameters for the optimal design to maximize the performance. A physical model for thermoelectric generators was introduced by Freunek et al. [20]. They presented geometric optimization and investigated the influence of Peltier heat leak conditions and load resistance on the thermoelectric power generation. The performance analysis of a two-stage thermoelectric generator was carried out by Chen et al. [21]. In the analysis, the fixed total number of thermoelectric elements of the combined device is considered and the allocations of the thermoelectric element pairs

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Nomenclature

A	area (m ²)	s	shape parameter of leg, dA/dx (m)
I	electrical current (A)	T	temperature (K)
k	thermal conductivity (W/mK)	W	power output (W)
K	overall thermal conductance of the thermoelectric generator (W/K)	Z	figure of merit (1/K)
L	length (height) of leg (m)	α	seebeck coefficient (V/K)
Q	heat flux (W)	μ	dimensionless shape parameter (Eq. (22))
R	overall electrical resistance of the thermoelectric generator (Ω)	η	efficiency
R_L	external load resistance (Ω)	σ	electrical conductivity (1/ Ω m)
		θ	temperature ratio (T_2/T_1)

among the two thermoelectric generators as well as the two thermoelectric heat pumps were optimized for maximizing the heat load and the coefficient of performance. The simulation study for thermoelectric power generation with multi-panels was carried out by Suzuki and Tanaka [22]. They indicated that the proper arrangements of the thermoelectric panels could shorten significantly the device area despite the fact that the output from the multi-panels could decrease a few percent. The solar thermoelectric generator for micro-power applications was investigated by Amatya and Ram [23]. They presented a thermodynamic analysis for predicting the thermal-to-electric conversion efficiency of the generator.

Although thermodynamic analysis of thermoelectric devices pertinent to the performance assessment has been presented extensively in the open literature [24–38], the influence of the geometric configuration of the thermoelectric device legs on the device performance is left obscure. Some of the earlier works [30,39,40] that consider the effect of the geometry on the performance of thermoelectric elements do not take into account the effect of geometry on the current that flows through the device. Consequently, in the present study the influence of thermoelectric device leg geometry on the power generation and the thermal efficiency that belongs to domain of “constructal design” [41] is examined. The theoretical analysis incorporating the operating parameters such as temperature ratio, external load, and device resistance, is presented. The functional relation between the thermoelectric power generation and the leg geometric parameter, namely shape parameter, is developed. The study is extended to include the efficiency analysis incorporating the leg shape parameter.

2. Thermal analysis

Consider the thermoelectric element of variable cross section shown in Fig. 1. The properties of leg material are assumed to be constant (no temperature or position dependence of the material properties, e.g. Seebeck coefficient, thermal and electrical conductivity) so the Thomson effect is inherently neglected. In addition, we assume no heat losses due to radiation and convection, no contacts (and contact resistances) and no external irreversibilities (transfer from heat sources) in the calculation. The calculation is in steady-state and based on a (quasi)-one dimensional approach. We use Dirichlet boundary conditions. Thus the efficiency of the thermoelectric power generator with legs of variable cross-section, which is shown in Fig. 1 is given as

$$\eta = \frac{I^2 R_L}{\alpha I T_1 + K(T_1 - T_2) - \frac{1}{2} I^2 R} \quad (1)$$

where K is the thermal conductance and R is the electrical resistivity of the thermoelectric generator.

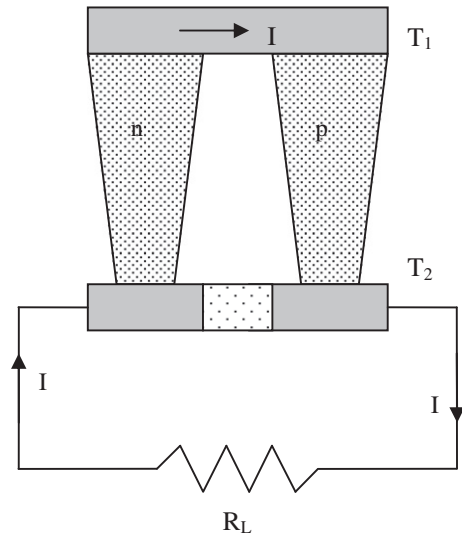


Fig. 1. Schematic view of a thermoelectric power generator with legs of variable cross-section.

The current I is a function of the net Seebeck coefficient $\alpha = \alpha_p - \alpha_n$ (the difference between the Seebeck coefficients of p and n junctions), the upper and lower junction temperatures (T_1 and T_2), the electrical resistance R and the external load resistance R_L as

$$I = \frac{\alpha(T_1 - T_2)}{R_L + R} \quad (2)$$

Substituting Eq. (2) in Eq. (1) the efficiency becomes

$$\eta = \frac{\alpha^2(T_1 - T_2)R_L}{K(R_L + R)^2 + \alpha^2 T_1(R_L + R) - \frac{1}{2} \alpha^2(T_1 - T_2)R} \quad (3)$$

The cross-sectional area of the thermoelectric generator leg shown in Fig. 2 can be written as

$$A(x) = A_0 + \left(x - \frac{L}{2}\right)s \quad (4)$$

where A_0 is the average (mid-height) cross-sectional area, L is the height of the leg, and s is the shape parameter of the leg, i.e. rate of change of the cross-sectional area of the legs along the height direction

$$s = \frac{dA}{dx}$$

Considering the likely manufacturing complexities, the shape parameter s is assumed to be independent from x (i.e., the cross-sectional area of the legs are assumed to vary linearly along the height direction) in the present work.

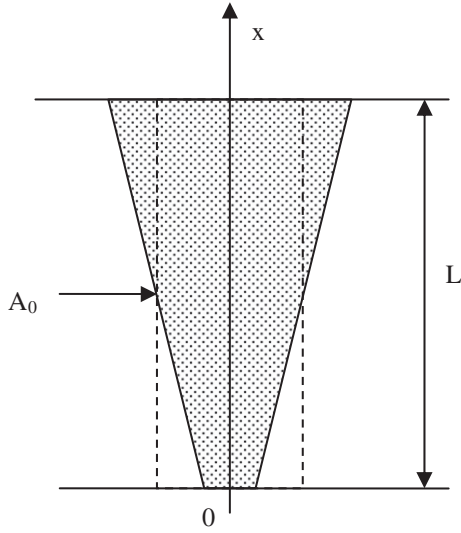


Fig. 2. Geometry of the thermoelectric generator leg.

The heat transfer rate through the leg along x is given by

$$\dot{Q} = -kA(x) \frac{dT}{dx} \quad (5)$$

After assuming a steady heating situation and isolated leg surfaces, Eq. (5) can be re-arranged as

$$\dot{Q} \int_0^L \frac{dx}{A(x)} = -k \int_{T_1}^{T_2} dT \quad (6)$$

Making use of Eq. (4) in Eq. (6), it and performing the integration

$$\dot{Q} = \frac{ks}{\ln \left(\frac{A_0 + s\frac{L}{2}}{A_0 - s\frac{L}{2}} \right)} (T_1 - T_2) \quad (7)$$

Eq. (7) indicates that the overall thermal conductance of the leg is

$$K_{\text{leg}} = \frac{ks}{\ln \left(\frac{A_0 + s\frac{L}{2}}{A_0 - s\frac{L}{2}} \right)} \quad (8)$$

Considering the two legs in Fig. 1, the total thermal conductance of the thermoelectric generator can be written as

$$K = \frac{(k_p + k_n)s}{\ln \left(\frac{A_0 + s\frac{L}{2}}{A_0 - s\frac{L}{2}} \right)} \quad (9)$$

where k_p and k_n are the thermal conductivities of the p -type and n -type legs, respectively.

On the other hand, the overall electrical resistance of the leg can be written as

$$R_{\text{leg}} = \int_0^L \frac{dx}{\sigma A(x)} \quad (10)$$

Substituting $A(x)$ from Eq. (4) and performing the integration, the overall electrical resistance is obtained as

$$R_{\text{leg}} = \frac{1}{\sigma s} \ln \left(\frac{A_0 + s\frac{L}{2}}{A_0 - s\frac{L}{2}} \right) \quad (11)$$

Similarly, considering the two legs the total electrical resistance of the thermoelectric generator becomes

$$R = \left(\frac{1}{\sigma_p} + \frac{1}{\sigma_n} \right) \frac{1}{s} \ln \left(\frac{A_0 + s\frac{L}{2}}{A_0 - s\frac{L}{2}} \right) = \frac{\sigma_p + \sigma_n}{\sigma_p \sigma_n s} \ln \left(\frac{A_0 + s\frac{L}{2}}{A_0 - s\frac{L}{2}} \right) \quad (12)$$

where σ_p and σ_n are the electrical conductivities of the p -type and n -type legs, respectively.

Substituting Eqs. (9) and (12) in Eq. (3) the efficiency of the thermoelectric generator can be written in dimensionless form as

$$\eta = (1 - \theta) \times \frac{2ZT_{\text{ave}} \left(1 + \sqrt{\frac{r_k}{r_\sigma}} \right)^2 \left(\frac{R_L}{R_0} \right)}{(1 + \theta) \left(\frac{K}{K_0} \right) \left(\frac{R_L}{R_0} + \frac{R}{R_0} \right)^2 + 2ZT_{\text{ave}} \left(1 + \sqrt{\frac{r_k}{r_\sigma}} \right)^2 \left[\frac{R_L}{R_0} + \frac{1}{2} \frac{R}{R_0} (1 + \theta) \right]} \quad (13)$$

where

$$\theta = \frac{T_2}{T_1}, \quad (\text{Temperature ratio}) \quad (14)$$

$$r_k = \frac{k_p}{k_n}, \quad (\text{Thermal conductivity ratio}) \quad (15)$$

$$r_\sigma = \frac{\sigma_p}{\sigma_n}, \quad (\text{Electrical conductivity ratio}) \quad (16)$$

$$ZT_{\text{ave}} = \frac{\alpha^2 \left(\frac{\sigma_n}{k_n} \right) T_1}{\left(1 + \sqrt{\frac{r_k}{r_\sigma}} \right)^2} \left(\frac{1 + \theta}{2} \right) \quad (\text{The figure of merit based on the average temperature}) \quad (17)$$

$$K_0 = \frac{A_0 k_n}{L} \quad (\text{Reference thermal conductance}) \quad (18)$$

and

$$R_0 = \frac{L}{A_0 \sigma_n} \quad (\text{Reference electrical resistivity}) \quad (19)$$

Thus, the overall thermal conductance and overall electrical resistivity can be written in dimensionless form, respectively, as

$$\frac{K}{K_0} = \frac{\mu(r_k + 1)}{\ln \left(\frac{1 + \mu/2}{1 - \mu/2} \right)} \quad (20)$$

and

$$\frac{R}{R_0} = \left(\frac{1 + r_\sigma}{r_\sigma} \right) \frac{\ln \left(\frac{1 + \mu/2}{1 - \mu/2} \right)}{\mu} \quad (21)$$

where μ is the dimensionless shape parameter defined as

$$\mu = \frac{sL}{A_0} \quad (22)$$

On the other hand, the power generation from the thermoelectric power generator is given as

$$\dot{W} = I^2 R_L \quad (23)$$

or

$$\dot{W} = \frac{\alpha^2 (T_1 - T_2)^2}{(R_L + R)^2} R_L \quad (24)$$

Thus the power generation can be written in dimensionless form as

$$\frac{\dot{W}}{K_0 T_2} = 2 \frac{(1 - \theta)^2}{\theta(1 + \theta)} \frac{ZT_{\text{ave}} \left(1 + \sqrt{\frac{r_k}{r_\sigma}} \right)^2 \left(\frac{R_L}{R_0} \right)}{\left(\frac{R_L}{R_0} + \frac{R}{R_0} \right)^2} \quad (25)$$

Table 1

Experimental measurements of the figure of merit for Bi_2Te_3 thermoelectric material at room temperature (300 K) as reported in the literature.

Fabrication method	Figure of merit (ZT) at 300 K
Co-evaporation [42]	0.93
Co-sputtering [43]	0.31
Metalorganic chemical vapor deposition (MOCVD) [44]	0.98
Co-evaporation [45]	0.80
Electrochemical deposition (ECD) [46]	0.07
Flash evaporation [47]	0.53
Cu I added (patent) [48]	0.69

The following parameters have been used in the analysis:

$$\frac{R_L}{R_0} = 10, ZT_{\text{ave}} = 1.5, r_k = 1.0 \text{ and } r_\sigma = 1.0$$

Table 1 shows experimental measurements of the figure of merit for Bi_2Te_3 thermoelectric material fabricated using various methods. As can be seen from the table an average figure of merit of $ZT = 0.8$ can be considered to represent the figure of merit of Bi_2Te_3 thermoelectric generator at room temperature (i.e. at 300 K). However, in our analysis, the average temperature in the thermoelectric element is used to evaluate the figure of merit (ZT_{ave}). Therefore, a representative value of $ZT_{\text{ave}} = 1.5$ is taken in the present work. Eqs. (13) and (25) are used to compute the device efficiency and the power generation with the shape parameter for various temperature ratios and external resistance ratio.

3. Results and discussion

A theoretical analysis of thermoelectric power generator is carried out and influence of generator leg geometric configuration on the efficiency and the power generation is examined. The leg area along the leg height is considered to change linearly while assuming constant leg thickness along the leg height. The area variation along the leg height is incorporated in the analysis.

Fig. 3 shows variation of efficiency with the shape parameter ($\mu = sL/A_0$) for different temperature ratios ($\theta = T_2/T_1$). It should be noted that $\mu > 0$ represents the increasing leg area with increasing leg height while $\mu < 0$ corresponds to decreasing leg area with increasing leg height. Fig. 4 shows three-dimensional view of efficiency variation with the shape parameter and temperature ratio. Efficiency reduces with decreasing shape parameter (μ) and becomes minimum for the shape parameter $\mu = 0$, which corresponds to a rectangular leg geometry. The increase in the efficiency of the thermoelectric power generator is associated with overall electrical resistance of the leg (Eq. (11)), which reduces with increasing values of the shape parameter. It should be noted that the shape parameter can vary between negative and positive values depending on the shape of the leg geometry. Increasing the shape parameter from zero ($\mu > 0$), efficiency of the thermoelectric generator increases. This is, again, due to reducing overall electric resistance with increasing the shape parameter. Although efficiency of the thermoelectric generator improves for $\mu > 0$ and $\mu < 0$ a care must be taken to account for the thermal stresses. In this case, reducing the cross-sectional area for $\mu > 0$ may result in high stress intensities in the leg at operating temperatures while limiting the life of the device. Moreover, increasing temperature ratio enhances the efficiency. This is true for all shape parameters. This is more pronounced for $-2 \leq \mu \leq -1$ and $1 \leq \mu \leq 2$. This indicates that non-linear effect of the shape parameter on the efficiency becomes considerable with increasing temperature ratio. Consequently, operating the thermoelectric power generator at high temperature ratio

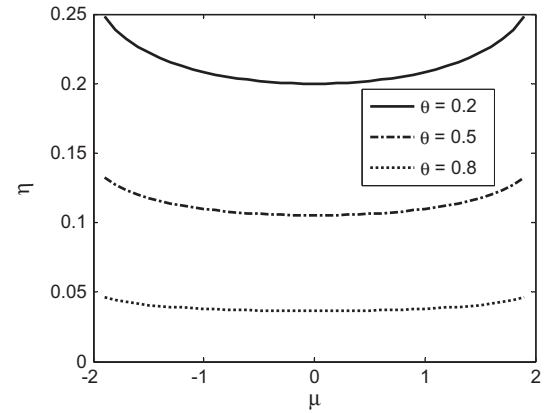


Fig. 3. Thermal efficiency variation with the shape parameter for different temperature ratios.

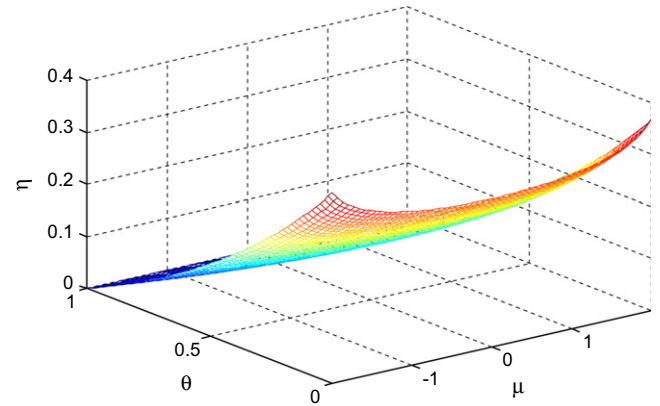


Fig. 4. Variation of efficiency with shape parameter and temperature ratio.

improves efficiency; however this improvement becomes considerable for the shape parameter in the range of $-2 \leq \mu \leq -1$ and $1 \leq \mu \leq 2$. Therefore trapezium shaped leg geometry results in better efficiency than that corresponding to the rectangular shape.

Fig. 5 shows thermoelectric generator output with the shape parameter. The influence of the shape parameter is notable on the thermoelectric power generation. This is more pronounced for low temperature ratio. The influence of shape parameter on the power generation is small for high temperature ratios, the power generation is also low for large temperature ratios. Consequently, the influence of the shape parameter on the power generation could not be observed clearly. On the other hand, for low temperature ratio, the influence of the shape parameter becomes considerable for both $-2 \leq \mu \leq -1$ and $1 \leq \mu \leq 2$. This is attributed to the value of the ratio of external load resistance (R_L) to the reference electrical resistance (R_0). Increasing R_L/R_0 reduces the effect of the shape parameter on the power generation. This can be seen from Fig. 6, in which the thermoelectric power with the shape parameter is given for $R_L/R_0 = 10$. Moreover, the power generation reduces with increasing or decreasing shape parameter, since the shape parameter enhances the overall electrical resistance of the thermoelectric generator (Eq. (12)). This results in low power generation (Eq. (25)). The power generated reaches its maximum for the shape parameter $\mu = 0$. This corresponds to rectangular leg geometry. Consequently, influence of the shape parameter could be minimized by increasing the external load parameter and the efficiency of the thermoelectric generator can be improved considerably for low temperature ratios.

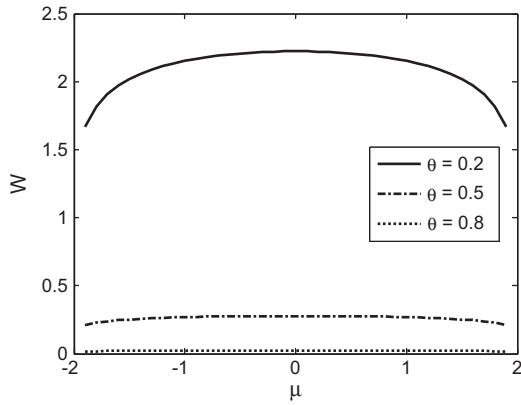


Fig. 5. Thermoelectric power generated as function of the shape parameter for different temperature ratios.

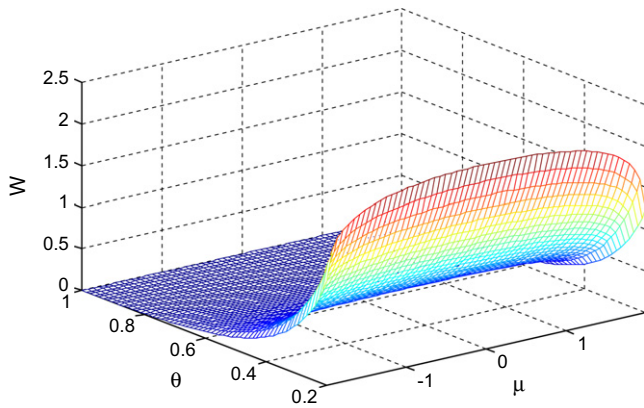


Fig. 6. Variation of thermoelectric power with shape parameter and temperature ratio.

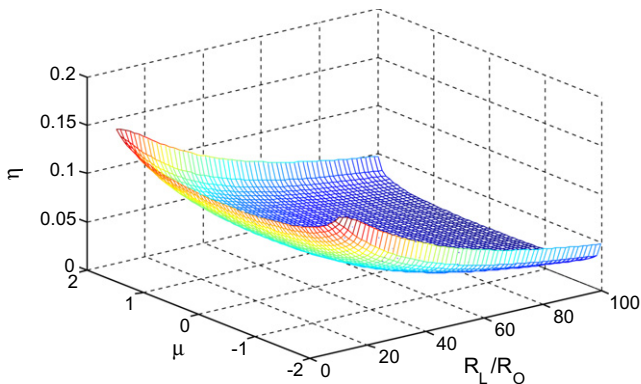


Fig. 7. Thermal efficiency of thermoelectric generator with the shape parameter and external load parameter.

Figs. 7 and 8 show three dimensional surface of the efficiency and the thermoelectric power generated for two R_L/R_O ratios. It is evident that increasing external load resistance reduces the efficiency and the power generated considerably. Therefore, selection of low external load resistance enhances both the efficiency and the power generation of the thermoelectric device. Moreover, the influence of the shape parameter on the efficiency and the power generation becomes notable for high temperatures (i.e. low temperature ratios). Therefore, the leg geometry modification only

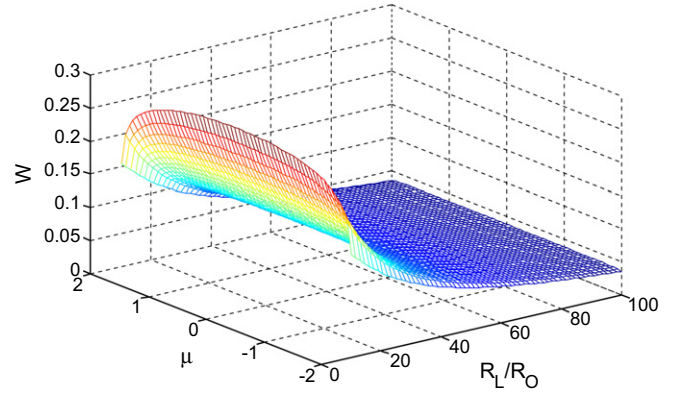


Fig. 8. Thermoelectric power generated as function of the shape parameter and external load parameter.

improves the efficiency when the device operates at low temperature ratios. Otherwise, this effect becomes negligible. The value of the power generation becomes maximum for the shape parameter for $\mu = 0$, which is true for all temperature ratios. Therefore, the shape parameter has an adverse affect on the power generation, but it has a favorable effect on the efficiency.

Fig. 9 shows the efficiency with the shape parameter for different values of the load parameter ratios. Efficiency increases for the shape parameter ranges $-2 \leq \mu \leq -1$ and $1 \leq \mu \leq 2$, which are

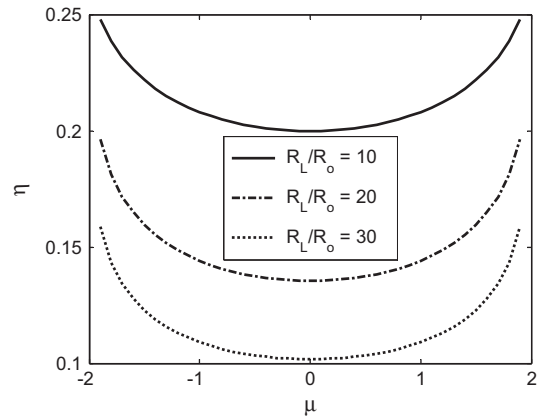


Fig. 9. Thermal efficiency variation with the shape parameter for different external load parameters.

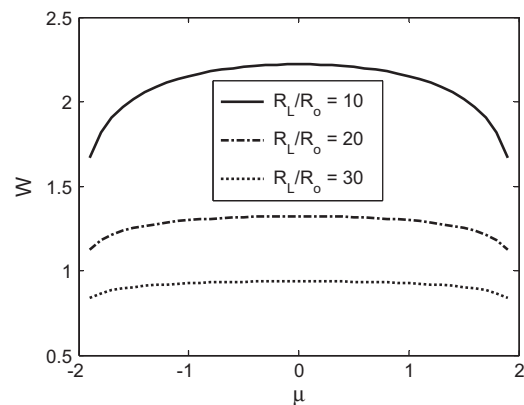


Fig. 10. Thermoelectric power generated as function of the shape parameter for different external load parameters.

similar to those shown in Fig. 3. This is associated with Eq. (13) in which, the efficiency is inversely related to the load parameter ratio. In this case, increasing external load resistance reduces the efficiency of the device, which is more pronounced for small values of the shape parameters. The efficiency reduces to its minimum regardless of the external load parameter ratio. Fig. 10 shows thermoelectric power generation with the shape parameter for different values of the load parameter ratio. The behavior of the thermoelectric power generation is similar to that is shown in Fig. 5. In this case, increasing load parameter ratio enhances the power generation, particularly for shape parameter $\mu = 0$. This behavior is attributed to Eq. (25), in which case, the shape parameter is inversely proportional to the thermoelectric power generation; further more, the influence of the load parameter ratio on this relation is more pronounced for $-2 \leq \mu \leq -1$ and $1 \leq \mu \leq 2$.

4. Conclusions

A theoretical analysis of thermoelectric power generator is carried out. The influence of the shape parameter, associated with the thermoelectric leg geometry, on the efficiency and power generation is formulated. The influence of the shape parameter on the efficiency and the power generation of the thermoelectric device are examined for different values of temperature ratio. It is found that the efficiency of the thermoelectric generator improves notably for certain ranges of the shape parameter, i.e. $-2 \leq \mu \leq -1$ and $1 \leq \mu \leq 2$. In this case, the leg geometry becomes a trapezium shape along the leg height. This is more pronounced for low temperature ratios. However, the power generation by the device reduces with increasing or decreasing shape parameter, which is more pronounced at low temperature ratio and the low external load resistance. Increasing external load resistance decreases the efficiency and the power generation. The influence of the shape parameter on the power generation becomes negligible as the external load resistance increase. Although the efficiency improves with increasing or decreasing shape parameter, the power generation reduces. Consequently, slight increase in the load resistance increases the device efficiency while slight reduction in the load resistance lowers the power generation. Therefore, when the high efficiency of the device is required decreasing or increasing in the shape parameter is favorable; however, when high power is required the shape parameter should be set to zero, which corresponds to the rectangular leg geometry.

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